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Some remarks on high derivative quantum gravity

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Abstract

We analyze the perturbative implications of the most general high derivative approach to quantum gravity based on a diffeomorphism invariant local action. In particular, we consider the super-renormalizable case with a large number of metric derivatives in the action. The structure of ultraviolet divergences is analyzed in some detail. We show that they are independent of the gauge fixing condition and the choice of field reparametrization. The cosmological counterterm is shown to vanish under certain parameter conditions. We elaborate on the unitarity problem of high derivative approaches and the distribution of masses of unphysical ghosts. We also discuss the properties of the low energy regime and explore the possibility of having a multi-scale gravity with different scaling regimes compatible with Einstein gravity at low energies. Finally, we show that the ultraviolet scaling of matter theories is not affected by the quantum corrections of high derivative gravity. As a consequence, asymptotic freedom is stable under those quantum gravity corrections.

Introduction

The formulation of a consistent theory of Quantum Gravity is still one of the major challenges in theoretical physics. One of the main fundamental problems is that there is no experimental evidence of any Quantum Gravity effect [1], whereas the classical theory covers from cosmology to current precision tests with great success. The situation is considerably more dramatic than for Quantum Electrodynamics at the end of the forties. The reason being that in the last case, although the theory was not completely consistent, there was experimental evidence on the existence of relativistic quantum effects associated to electron dynamics. Nature provided confidence on the co-existence of special relativity and quantum mechanics although field theory was not yet at sight. An analogous conviction does not exist

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in Quantum Gravity. On the other hand since the quantum effects can only, in principle, be observed at the Plank scale which lies too far from current experiments, the lack of experimental data is not unnatural. Consequently, any theoretical model of Quantum Gravity has to be necessarily based on theoretical principles without phenomenological constraints.

On spite of this freedom of the theory of Quantum Gravity, the construction of consistent models meets very serious difficulties. First, the quantum theory based on the Einstein-Hilbert action is non-renormalizable [2, 3]. The radiative corrections to the effective action contain divergences which involve a number of derivatives of the metric in the counterterms which is increasing with the number of loops. Indeed, one can use the general theorems on covariant renormalization (see [4] and references therin) to show that all these counterterms are general covariant local expressions. Thus, it is possible to remove all the divergences starting from the Einstein theory by adding an infinite tower of possible higher derivative terms, regarded as the perturbations. In that case the definition of the renormalized theory requires, in general, the introduction of an infinite number of renormalization conditions, unless some parameter reduction mechanism appears [5]. Recently, it was pointed out [6] that some large distances effects remain, however, independent of the renormalization conditions involving higher order terms of the effective action. In particular this occurs for the quantum corrections to the gravitational Newtonian potential. However, the analysis of other effects like the scaling dependence of the cosmological and gravitational constants, depends on the concrete formulation of the quantum theory.

One can always imagine that the extremely high energy UV regime is described by qualitatively different theory like string theory, free of renormalizability problems, which will provide a natural reduction parameter scheme at intermediate scales. Having this as perspective, field theoretical approaches to quantum gravity should be considered as effective field theories. However the predictions of string theory quantum gravity effects at the intermediate energies still remain unveiled. On the other hand, the string effective action is well defined only on shell. Off shell continuations are not uniquely defined because of reparametrization invariance [7]–[15]. For such a reason, we will consider the most general effective action² which can be generated by string theory [17]

$$S_{\text{eff}}(g_{\mu\nu}) = \lim_{N \rightarrow \infty} \sum_{n=0}^{N+2} \alpha_{2n} m^{4-2n} \int d^4x \sqrt{-g} \mathcal{O}_{2n}(\partial_\lambda g_{\mu\nu}), \quad (1)$$

where m^2 is the only dimensional parameter of our fundamental theory (in string theory it is the string tension $1/\alpha'$) while $\mathcal{O}_{2n}(\partial_\lambda g_{\mu\nu})$ denotes the general covariant scalar terms containing $2n$ derivatives of the metric $g_{\mu\nu}$ and the constants α_n are dimensionless couplings. Although theories with higher derivatives like (1) are in general non-unitary at the quantum level, string theory is both unitary and renormalizable. In particular, one can always

²We do not consider the dilaton and the antisymmetric fields for the sake of simplicity throughout this paper. The inclusion of such fields does not introduce qualitative changes in the results.

choose a special parametrization without unphysical ghosts [7, 8], [14], although from a pure string point of view there are no means to distinguish this special parametrization of the metric except for the absence of ghosts [9] (see also [16]). However if one considers an approximation to the effective theory and makes a truncation of the series (1), the unitarity problem reappears. The problem shows up even at the classical level where, because of the existence of the unphysical ghosts with negative energy, it leads to classical instabilities [18]. The quantum unitarity problem has been considered in great detail in the particular case of (four derivative) R^2 -gravity [19, 3], [20]-[29]. In that case, the ultraviolet behaviour of the propagators and vertices leads to a renormalizable theory [20, 4]. Unfortunately the particle content of this theory contains, besides the massless graviton, a spin-2 massive unphysical ghost which violates unitarity [20, 18]. Despite a lot of interesting attempts to solve the unitarity problem in R^2 -gravity [21]–[24] it turns out that the massive ghost is not removed by radiative corrections, and therefore high derivative theory can not be considered as the fundamental theory of quantum gravity³. The appearance of the massive ghosts is the price to pay for renormalizability. Their contributions are essential to reduce the dimension of counterterms. However, since the masses of the ghosts are of the order of the Planck mass, the fourth derivative quantum gravity can be successfully used as an effective theory at the energies below this scale, where the unphysical ghosts are not generated [26]-[29].

The aim of the present paper is to extend those results for the general case of high derivative gravity. We will truncate the effective string theory action S_{eff} taking a finite value for N . If $N > 0$ the corresponding theory contains more than four derivatives of the metric and it becomes super-renormalizable. This provides a natural framework to study the possibility of different scaling regimes compatible with Einstein gravity at low energies and different scenarios for the ultraviolet behaviour of the cosmological constant.

The organization of the paper is as follows. In section 2 we consider the quantization of the general higher derivative theory and show its super-renormalizability. In section 3 the general structure of ultraviolet divergences is analyzed and the cosmological counterterm is explicitly calculated for the general case. We also study the gauge fixing independence of the counterterms. The low energy regime of the theory is examined and shown to be equal to that of Einstein gravity in section 4, where we also discuss the spectrum of massive ghosts, and the possibilities of having different scaling regimes. In section 5 we study the coupling of high derivative gravity to general gauge theories and show that their beta functions are not modified by the gravitational corrections. Finally, a discussion of the results is developed in section 6, and some technical aspects are postponed to three appendices.

2. Gauge fixing, quantization and power counting.

³It is a remarkable fact that all attempts to derive a Field Theory of Gravity from non-commutative geometry lead to R^2 -gravity

The first two terms of the action S_{eff} are the cosmological term

$$S_{\text{eff}}^{(0)}(g_{\mu\nu}) = m^4 \alpha_0 \int d^4x \sqrt{g}$$

and the Hilbert-Einstein action,

$$S_{\text{eff}}^{(2)}(g_{\mu\nu}) = m^2 \alpha_2 \int d^4x \sqrt{g} R$$

The third term ($n = 2$) leads to the mentioned fourth derivative gravity

$$S_{\text{eff}}^{(4)}(g_{\mu\nu}) = \int d^4x \sqrt{g} (\alpha_4^1 R_{\alpha\beta} R^{\alpha\beta} + \alpha_4^2 R^2)$$

(see [30] for an introduction and more complete references). Higher order terms ($n > 2$) can be expressed in terms of the Riemann curvature tensor, Ricci tensor, scalar curvature and their covariant derivatives. For $n = 3$ we have two different types of terms $R^{(3)}$ and $\nabla R \cdot \nabla R$ and for $n = 4$ we have R^4 , $R \nabla R \cdot \nabla R$, $\nabla^2 R \cdot \nabla^2 R$. N -th order include terms from R^{N+2} and $R^{N-2} \nabla R \cdots \nabla R$ to $R \cdots \square^N R$. The dots indicate all possible contractions of tensor indices.

Perturbation theory is generated by the standard expansion around the flat metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}.$$

The contributions of the $N > 0$ terms to the propagator can only come from terms of second order in the curvature. Terms of third and higher orders in the curvature contribute only to vertices, because they involve more than two $h_{\mu\nu}$ fields. Using Bianchi identities

$$\nabla_\lambda R_{\mu\nu\alpha\beta} + \nabla_\beta R_{\mu\nu\lambda\alpha} + \nabla_\alpha R_{\mu\nu\beta\lambda} = 0 \quad (2)$$

one can easily reduce the terms of second order in the curvature (with total $2n$ derivatives) to the form

$$\alpha_n^1 R_{\alpha\beta} \square^{n-2} R^{\alpha\beta} + \alpha_n^2 R \square^{n-2} R + \alpha_n^3 R_{\mu\nu\alpha\beta} \square^{n-2} R^{\mu\nu\alpha\beta} \quad (3)$$

Now one can use (2) again as it was done in [8] for the $N = 1$ (six-derivative) terms to get

$$\begin{aligned} R_{\mu\nu\alpha\beta} \square^{n-2} R^{\mu\nu\alpha\beta} &= -\nabla_\lambda R_{\mu\nu\alpha\beta} \square^{n-2} \nabla^\lambda R^{\mu\nu\alpha\beta} + \mathcal{O}(R^3) + \nabla_\mu \Omega^\mu \\ &= 4R_{\alpha\beta} \square^{n-2} R^{\alpha\beta} - R \square^{n-2} R + \mathcal{O}(R^3) + \nabla_\mu \Omega'^\mu \end{aligned} \quad (4)$$

This is indeed similar to the relation which takes place in the fourth derivative gravity because of the topological nature of the Gauss-Bonnet term [2]. Of course, for any $n \neq 2$ the term

$$\int d^4x \sqrt{-g} (R_{\mu\nu\alpha\beta} \square^{n-2} R^{\mu\nu\alpha\beta} - 4R_{\alpha\beta} \square^{n-2} R^{\alpha\beta} + R \square^{n-2} R) \quad (5)$$

is not topological and gives rise to non-trivial contributions to the vertices. Since the $\mathcal{O}(R^3)$ terms do not contribute to the propagator, the relevant terms of order $n > 2$ can be written as

$$\alpha_n^1 R_{\alpha\beta} \square^{n-2} R^{\alpha\beta} + \alpha_n^2 R \square^{n-2} R \quad (6)$$

If $\alpha_{2N+4}^1 \neq 0$ and $\alpha_{2N+4}^1 + 3\alpha_{2N+4}^2 \neq 0$, the highest order terms of (6) are nondegenerate once we introduce a gauge fixing and asymptotically they behave like $\mathcal{O}(k^{2n+4})$ for large momenta. The gauge fixing condition can be introduced within the standard Faddeev-Popov prescription. In order to improve the regularity properties of the quantum fields it is convenient to add a higher derivative covariant operator for the longitudinal modes in the gauge fixing condition [20, 30] (see also [31]–[33] for a general discussion on gauge theories)

$$S_{\text{gf}} = \int d^4x \chi_\mu C^{\mu\nu} \chi_\nu \quad (7)$$

where

$$\begin{aligned} \chi_\mu &= \partial_\lambda h_\mu^\lambda - \beta \partial_\mu h_\lambda^\lambda \\ C^{\mu\nu} &= - \sum_{n=2}^{N+2} [\sigma_n g^{\mu\nu} \square + (\gamma_n - 1) \nabla^\mu \nabla^\nu] \left(\frac{\square}{m^2} \right)^{n-2} \end{aligned} \quad (8)$$

$\beta, \sigma_n, \gamma_n$ being dimensionless gauge fixing parameters. Regardless to the number of derivatives, the propagator of the quantum metric can be written in terms of the irreducible spin 2, 1, 0 projectors and some spin-zero transfer operators [20]. By choosing the gauge fixing parameters $\beta, \sigma_n, \gamma_n$ in a special way one can always remove all the spin-1 states from the spectrum. The spin-2 states are gauge-fixing independent, just as in the fourth derivative gravity, and if $\alpha_{2N+4}^1 \neq 0$, the propagator of the spin two states has the asymptotic ultraviolet behaviour $\mathcal{O}(k^{-(2N+4)})$. The same occurs for the propagator of spin-0 states but there the condition reads $\alpha_{2N+4}^1 + 3\alpha_{2N+4}^2 \neq 0$. Throughout we shall assume that both conditions are satisfied and therefore the propagator of the quantum metric behaves like $\mathcal{O}(k^{-(2N+4)})$ for large momenta. One can absorb the determinant of the covariance operator $C^{\mu\nu}$ into the action of Faddeev-Popov ghosts [26],

$$S_{FP} = \int d^4x \sqrt{-g} \left[\bar{c}^\lambda C_{\mu\nu} \nabla^\nu (\nabla^\mu c_\lambda + \nabla_\lambda c^\mu) - 2\beta \bar{c}^\lambda C_{\lambda\nu} \nabla^\nu (\nabla_\mu c^\mu) \right] \quad (9)$$

and get that their propagator also behaves like $\mathcal{O}(k^{-(2N+4)})$ in the UV regime. The extra contribution of the operator C into the ghost sector has to be compensated by the corresponding power of the determinant $\det C^{\mu\nu}$ [26] (see also [31]–[33] for a general discussion on gauge theories). In summary, the partition function is given by

$$\int [\delta h_{\mu\nu}] [\delta c_\tau] [\delta \bar{c}^\lambda] (\det C^{\mu\nu})^{-\frac{1}{2}} e^{iS(g_{\mu\nu}) + iS_{\text{gf}} + iS_{FP}} \quad (10)$$

With that gauge fixing choice the propagators of both metric and ghosts have the same asymptotic behaviour in the UV limit $\mathcal{O}(k^{-(2N+4)})$. Unfortunately the interaction vertices pick up all possible number of derivatives from zero to $2N + 4$, in such a way that one loop divergences remain unregularized. Indeed, if we evaluate the superficial degree of divergency D of an arbitrary p -loop diagram with n_{2r} vertices with $2r$ derivatives $r = 0, 1, \dots, N + 2$ we get

$$D + d_{\text{ext}} = 4 + 2N - 2Np - \sum_{r=0}^{N+1} (2N + 4 - 2r) n_{2r} \quad (11)$$

where d_{ext} is the total number of derivatives acting on external lines. From the power counting identity (11) it follows that divergent graphs satisfy the inequality

$$d_{\text{ext}} \leq 4 + 2N - 2Np \quad (12)$$

which implies that divergences can only appear for higher loops diagrams with $N = 0, 1, 2$. For $N > 2$ divergences appear only in the one-loop diagrams. For any $N > 0$ the divergent terms involve less powers of the curvature than those bare action, and moreover, this power is decreasing with the loop order. We remark that the covariance of all the counterterms (in an invariant regularization) is guaranteed by the general theorems [4] which can be trivially generalized for diffeomorphism invariant theories of the form S_{eff} .

Thus, for any finite $N > 0$, the structure of divergences becomes simpler, the theory is super-renormalizable. For $N > 2$ the situation is even better because all the divergences appear at one loop order of perturbation theory. Now, in any case the structure of one loop divergences is not changed. This is very similar to what happens in gauge theories where the method of high covariant derivatives does not smooths the behaviour of one loop contributions [31].

If $N > 2$, since the only UV divergences appear in one loop diagrams, they can be removed by Pauli-Villars determinant regulators using the methods first introduced for gauge theories. In such a case we end up with a completely finite theory (all beta functions vanish) and the corresponding theory may be considered as a complete regularization of Quantum Gravity. This is nothing but the implementation for quantum gravity of the high derivative method introduced by Slavnov [34]. In such a regularization one can look for non-perturbative effects and specially for non-trivial fixed points where the theory might be unitary and renormalizable. If such fixed points would exist most of the technical problems mentioned in the introduction will be overcome.

For any $N > 0$ the theory is super-renormalizable, because the local covariant counterterms have less derivatives than the classical action and the coefficients of the terms with more derivatives do not need any kind of infinite renormalization. Thus, the bare values of those coefficients can be kept finite. Their explicit value can only be fixed by looking at the

phenomenological implications of the theory or the predictions of the fundamental theory which generates our theory as low energy effective theory. The only sector of the theory which is subject of infinite renormalization is of the form

$$\int d^4x \sqrt{-g} \left\{ \alpha_4^1 R_{\alpha\beta} R^{\alpha\beta} + \alpha_4^2 R^2 + \alpha_2 m^2 R + \alpha_0 m^4 \right\} + (\text{surface terms}) \quad (13)$$

After renormalization the scaling behaviour of the physical quantities is governed by a renormalization group equation. Since the only parameters which undergo an infinite renormalization are those contained in the lower derivative part (13) of the action, the scale dependence of the effective theory S_{eff} is encoded in the beta-functions for the four parameters $\beta_{\alpha_4^1}$, $\beta_{\alpha_4^2}$, β_{α_2} , β_{α_0} . We shall focus on the evaluation of these beta-functions.

Indeed for large N the Feynman rules are rather involved and in general even one loop calculations are very difficult. In the next section we develop a method to calculate these beta-functions in one-loop approximation for the most general case, and we shall perform an explicit calculation of the cosmological constant beta function $\beta_\Lambda = \beta_{\alpha_0}$. Our method is essentially based on the techniques introduced by Barvinsky and Vilkovisky [37], which can be, in principle, applied for the calculation of $\beta_{\alpha_4^1}$, $\beta_{\alpha_4^2}$, β_{α_2} as well. The only difficulty is that the manipulation of algebraic expressions becomes much more involved in those cases.

3. One-loop results.

Before entering into the calculation of beta-functions, let us show that they are independent of the choice of the gauge fixing parameters (8) and the parametrization of quantum gravitational fields. Local counterterms can be, in principle, gauge and parametrization dependent, but we will show that these dependence vanishes for high derivative gravity. This follows from the fact that for $N > 2$ the only UV divergences appear at one loop order and the explicit relation which exists between the divergent one-loop counter-terms associated to two different sets of the gauge fixing parameters $\sigma = (\sigma_n, \beta, \gamma_n)$ and $\bar{\sigma} = (\bar{\sigma}_n, \bar{\beta}, \bar{\gamma}_n)$ [28, 35] (see also Appendix B for a selfcontained derivation).

$$\Gamma_{\text{div}}(\sigma) - \Gamma_{\text{div}}(\bar{\sigma}) = \int d^4x \sqrt{-g} \frac{\delta S}{\delta g_{\mu\nu}} \Sigma_{\mu\nu}(g_{\alpha\beta}, \sigma, \bar{\sigma}), \quad (14)$$

$\Sigma_{\mu\nu}(g_{\alpha\beta}, \sigma, \bar{\sigma})$ being some local function of metric and gauge fixing parameters. Power counting tells us that for any choice of the gauge fixing parameters the divergent counterterms are local expressions with up to four derivatives of the metric. Thus, the left hand side of the identity (14) can only contain such a type of terms. But the right hand side which is proportional to classical motion equations contains terms with $2N+4$ derivatives. Hence the equality can hold if only and only if $\Sigma_{\mu\nu}(g_{\alpha\beta}, \sigma, \bar{\sigma})$ is identically zero. One-loop divergences are, therefore, gauge independent. Since $N > 2$ there are not more divergences in higher loops and, therefore, we have proved that all the beta-functions $\beta_{\alpha_4^1}$, $\beta_{\alpha_4^2}$, β_{α_2} , β_{α_0} do not

depend on the gauge fixing condition. In fact, by means of Nielsen identities [36] it can be proved that even in the case $0 < N \leq 2$ there is no dependence on the gauge fixing parameters in the divergent counter-terms generated by higher loops. The independence on the parametrization of the quantum metric can be proved in a similar way because two different reparametrizations lead to divergent contributions which satisfy an equation similar to (14).

Let us now calculate the one-loop radiative corrections. The leading ultraviolet term is given by

$$\alpha_{2N+4}^1 R_{\alpha\beta} \square^N R^{\alpha\beta} + \alpha_{2N+4}^2 R \square^N R \quad (15)$$

We shall assume that $\alpha_{2N+4}^1 \neq 0$ and $\alpha_{2N+4}^1 + 3\alpha_{2N+4}^2 \neq 0$. In the background field method the metric is split into a background metric $g_{\mu\nu}$ and quantum metric $h_{\mu\nu}$,

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}.$$

We introduce the background field gauge fixing condition

$$S_{\text{gf}} = \int d^4x \sqrt{-g} \chi_\mu C^{\mu\nu} \chi_\nu \quad (16)$$

where

$$\chi_\mu = \nabla_\lambda h_\mu^\lambda - \beta \nabla_\mu h$$

$$C^{\mu\nu} = -\frac{1}{\alpha} (g^{\mu\nu} \square + \gamma \nabla^\mu \nabla^\nu - \nabla^\nu \nabla^\mu) \left(\frac{\square}{m^2} \right)^N \quad (17)$$

which is a covariant generalization of (7) defined by replacing ordinary derivatives by covariant derivatives with respect to the background metric $g_{\mu\nu}$. The coefficients of the gauge fixing condition α, β, γ are arbitrary parameters. Many of the coefficients of (8) have been set equal to zero to simplify the calculations, but the divergences will not depend on their values. The one-loop effective action is

$$\Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln H_{\mu\nu,\alpha\sigma} - i \text{Tr} \ln M_\alpha^\sigma - \frac{i}{2} \text{Tr} \ln C^{\mu\nu} \quad (18)$$

where

$$H_{\mu\nu,\alpha\sigma} = \frac{\delta^2 S_{\text{eff}}}{\delta h_{\alpha\sigma} \delta h_{\mu\nu}} \Big|_{h=0} + \frac{\delta \chi_\lambda}{\delta h_{\alpha\sigma}} C^{\lambda\tau} \frac{\delta \chi_\tau}{\delta h_{\mu\nu}} \Big|_{h=0} \quad (19)$$

and M_α^σ is the operator of the *FP* ghost action

$$M_\alpha^\sigma = \square \delta_\alpha^\sigma + \nabla_\alpha \nabla^\sigma - 2\beta \nabla^\sigma \nabla_\alpha. \quad (20)$$

The contributions of the operators M_α^σ and $C^{\mu\nu}$ to the divergences of the effective action (18) can be easily evaluated by means of Barvinsky-Vilkovisky techniques [37]. The result is

$$-i \text{Tr} \ln M_\mu^\nu = -\frac{1}{3\varepsilon} \int d^4x \sqrt{-g} \left[\left(-\frac{4}{5} + \frac{1}{8\beta^2} + \frac{1}{2\beta} \right) R_{\tau\lambda} R^{\tau\lambda} + \left(\frac{3}{5} + \frac{1}{16\beta^2} \right) R^2 \right] \quad (21)$$

and,

$$\begin{aligned} -\frac{i}{2} \text{Tr} \ln C_\mu^\nu &= -\frac{iN}{2} \text{Tr} \ln (\delta_\mu^\nu \square) - \frac{i}{2} \text{Tr} \ln (g^{\mu\nu} \square + \gamma \nabla^\mu \nabla^\nu - \nabla^\nu \nabla^\mu) = \\ &= -\frac{1}{3\varepsilon} \int d^4x \sqrt{-g} \left[\left(-\frac{4N}{5} + \frac{7}{10} \right) R_{\tau\lambda} R^{\tau\lambda} + \left(\frac{7N}{20} - \frac{3}{20} \right) R^2 \right]. \end{aligned} \quad (22)$$

We remark that the last expression does not depend on γ due to the cancellation first pointed out in Ref. [26].

The main technical problem is the calculation of $\text{Tr} \ln H_{\mu\nu,\alpha\sigma}$. H is a differential operator of $(2N+4)$ order, with coefficients depending on the curvature tensor of the background metric and its derivatives, the parameters of the gravitational action, and the gauge fixing parameters α, β, γ . In spite of these difficulties one can derive a general formula for the divergent part of $\text{Tr} \ln H_{\mu\nu,\alpha\beta}$ and then perform an explicit calculation for the cosmological counterterm for arbitrary value N . Let us introduce dimensionful coupling constants

$$\omega_n^i = m^{-2n} \alpha_{2n+4}^i$$

by absorbing the mass parameter in the dimensionless couplings α_n^i of S_{eff} . Since the divergences do not depend on the explicit values of the gauge fixing parameters, it is very convenient to fix their values to simplify the calculations. A convenient choice is

$$\alpha = \frac{2}{\omega_N^1}, \quad \gamma = -\frac{2\omega_N^2}{\omega_N^1}, \quad \beta = \frac{\omega_N^1}{4\omega_N^2} + 1. \quad (23)$$

For this choice of the gauge fixing, the operator becomes

$$\begin{aligned} H_{\mu\nu,\alpha\beta} &= \left(\frac{\omega_N^1}{4} \delta_{\mu\nu}^{\rho\sigma} - \frac{\omega_N^1 (\omega_N^1 + 4\omega_N^2)}{16\omega_N^2} g_{\mu\nu} g^{\rho\sigma} \right) \left\{ \delta_{\rho\sigma,\alpha\beta} \square^{N+2} \right. \\ &\quad + V_{\rho\sigma,\alpha\beta}^{\lambda_1 \lambda_2 \dots \lambda_{2N+2}} \nabla_{\lambda_1} \nabla_{\lambda_2} \dots \nabla_{\lambda_{2N+2}} + W_{\rho\sigma,\alpha\beta}^{\tau_1 \tau_2 \dots \tau_{2N+1}} \nabla_{\tau_1} \nabla_{\tau_2} \dots \nabla_{\tau_{2N+1}} \\ &\quad \left. + U_{\rho\sigma,\alpha\beta}^{v_1 v_2 \dots v_{2N}} \nabla_{v_1} \nabla_{v_2} \dots \nabla_{v_{2N}} + \mathcal{O}(\nabla^{2N-1}) \right\} \end{aligned} \quad (24)$$

where V, W, U depend on the dimensionful ratios like $\omega_{N-1}^1/\omega_N^1$ and on the curvature tensor of the background metric and its covariant derivatives. Now we can use the Barvinsky-Vilkovisky method [37] to extract the divergences of $\text{Tr} \ln H_{\mu\nu,\alpha\beta}$. The pre-factor of (24) does not contribute to the divergences and therefore it can be omitted. Hence

$$\text{Tr} \ln H_{\mu\nu,\alpha\beta} = (N+2) \text{Tr} \ln [\delta_{\mu\nu,\alpha\beta} \square] + \text{Tr} V_{\mu\nu,\alpha\beta}^{\lambda_1 \lambda_2 \dots \lambda_{2N+2}} \nabla_{\lambda_1} \nabla_{\lambda_2} \dots \nabla_{\lambda_{2N+2}} \frac{1}{\square^{N+2}}$$

$$\begin{aligned}
& + \text{Tr } W_{\mu\nu,\alpha\beta}^{\tau_1\tau_2\cdots\tau_{2N+1}} \nabla_{\tau_1} \nabla_{\tau_2} \cdots \nabla_{\tau_{2N+1}} \frac{1}{\square^{N+2}} + \text{Tr } U_{\mu\nu,\alpha\beta}^{v_1 v_2 \cdots v_{2N}} \nabla_{v_1} \nabla_{v_2} \cdots \nabla_{v_{2N}} \frac{1}{\square^{N+2}} \\
& - \frac{1}{2} \delta^{\rho\sigma,\delta\phi} \text{Tr } V_{\mu\nu,\rho\sigma}^{\lambda_1 \cdots \lambda_{2N+2}} \nabla_{\lambda_1} \cdots \nabla_{\lambda_{2N+2}} \frac{1}{\square^{N+2}} V_{\delta\phi,\alpha\beta}^{\lambda'_1 \cdots \lambda'_{2N+2}} \nabla_{\lambda'_1} \cdots \nabla_{\lambda'_{2N+2}} \frac{1}{\square^{N+2}} \\
& + (\text{terms of higher background dimension}) \tag{25}
\end{aligned}$$

Indeed by dimensional arguments the last higher background dimension terms do not contribute to the divergences. The only term which can not be directly handled by the method of [37], is that which involves two V matrices. In particular there are new terms coming from the commutation of V with the covariant derivatives and the operators \square^{-1} . However, since V has dimensions of curvature, and derivatives of V increases its dimension, those terms do not contribute to the divergent part of the effective action. Therefore as far as one loop divergences are concerned V may be considered as a constant and we can ignore its derivatives. The corresponding contribution can be written as ⁴

$$-\frac{1}{2} \delta^{\nu\psi,\delta\phi} \text{Tr } V_{\mu\nu\nu\psi}^{\lambda_1 \cdots \lambda_{2N+2}} V_{\delta\phi\alpha\beta}^{\lambda'_1 \cdots \lambda'_{2N+2}} \nabla_{\lambda_1} \cdots \nabla_{\lambda_{2N+2}} \nabla_{\lambda'_1} \cdots \nabla_{\lambda'_{2N+2}} \frac{1}{\square^{2N+4}} \tag{26}$$

Now all the terms in the expansion admit the direct substitution of the universal trace formulae of Barvinsky-Vilkovisky and we can derive a general formula for the one-loop divergences of the theory. To apply this formula for a general N one needs to expand the action up to second order in quantum fields, keeping the first and second orders in background curvature while neglecting the higher orders and derivatives of the curvature. The algebra involved in the calculation is quite cumbersome. We will restrict ourselves to the calculation of the divergent contributions to the cosmological constant counterterm. In order to simplify the calculations we remark that the background field method gives the result which is valid for any background metric, including the flat one. Thus the cosmological counterterm can be derived for the flat background metric, which considerably simplifies the calculation. According to (18) and (19) the one-loop divergences are related with the Hessian of the classical action. If the background metric is flat, one can, therefore, ignore all the terms which have more than two powers of curvature, and calculate the divergences of the theory

$$S = \int d^4x \sqrt{-g} \left[\sum_{n=0}^N \left(\omega_n^1 R_{\mu\nu} \square^n R^{\mu\nu} + \omega_n^2 R \square^n R \right) - \omega_{-1} R + \omega_{-2} \right]. \tag{27}$$

The cosmological constant term $\omega_{-2}\sqrt{-g}$ does not give divergent contribution for any $N > 0$, and Einstein term is relevant for $N = 1$ only. Let us first consider the case of $N > 1$. The relevant part of operator H for the divergent contribution has the form

$$H_{\mu\nu,\alpha\beta} = \delta^{\mu\nu,\alpha\beta} \square^{N+2} + V_{\mu\nu,\alpha\beta}^{\lambda\tau\rho\sigma} \square^{N-1} \nabla_\lambda \nabla_\tau \nabla_\rho \nabla_\sigma + U_{\mu\nu,\alpha\beta}^{\lambda\tau\rho\sigma} \square^{N-2} \nabla_\lambda \nabla_\tau \nabla_\rho \nabla_\sigma \tag{28}$$

⁴ By the same dimensional reasons the matrix W of (24) can not generate divergences and for such a reason we shall not consider its contribution.

Using well known expansions of $R_{\mu\nu}$ we obtain the following expressions for V

$$\begin{aligned} [V_{\mu\nu,\alpha\beta}]^{\lambda\tau\rho\sigma} &= \frac{\omega_{N-1}^1}{\omega_N^1} \delta_{\mu\nu,\alpha\beta} g^{\lambda\tau} g^{\rho\sigma} + \frac{\omega_N^1 \omega_{N-1}^2 - \omega_N^2 \omega_{N-1}^1}{\omega_N^1 (\omega_N^1 + 3\omega_N^2)} g_{\mu\nu} g_{\alpha\beta} g^{\lambda\tau} g^{\rho\sigma} \\ &+ \frac{2(\omega_{N-1}^1 + 2\omega_{N-1}^2)}{\omega_N^1} \delta_{\mu\nu,\alpha\beta} \delta^{\lambda\tau,\rho\sigma} + \frac{\omega_N^2 \omega_{N-1}^1 - \omega_N^1 \omega_{N-1}^2}{\omega_N^1 (\omega_N^1 + 3\omega_N^2)} g_{\mu\nu} \delta_{\alpha\beta}^{\lambda\tau} g^{\rho\sigma} \\ &- \frac{\omega_{N-1}^1}{2\omega_N^1} g^{\lambda\tau} (g_{\mu\alpha} \delta_\beta^\sigma \delta_\nu^\rho + g_{\nu\alpha} \delta_\beta^\sigma \delta_\mu^\rho + g_{\mu\beta} \delta_\alpha^\sigma \delta_\nu^\rho + g_{\nu\beta} \delta_\alpha^\sigma \delta_\mu^\rho) \\ &- \frac{\omega_{N-1}^1 + 4\omega_{N-1}^2}{\omega_N^1} g_{\lambda\tau} g_{\alpha\beta} \delta_{\nu\mu}^{\rho\sigma} \end{aligned} \quad (29)$$

and for U

$$\begin{aligned} [U_{\mu\nu,\alpha\beta}]^{\lambda\tau\rho\sigma} &= \frac{\omega_{N-2}^1}{\omega_N^1} \delta_{\mu\nu,\alpha\beta} g^{\lambda\tau} g^{\rho\sigma} + \frac{\omega_N^1 \omega_{N-2}^2 - \omega_N^2 \omega_{N-2}^1}{\omega_N^1 (\omega_N^1 + 3\omega_N^2)} g_{\mu\nu} g_{\alpha\beta} g^{\lambda\tau} g^{\rho\sigma} \\ &+ \frac{2(\omega_{N-2}^1 + 2\omega_{N-2}^2)}{\omega_N^1} \delta_{\mu\nu,\alpha\beta} \delta^{\lambda\tau,\rho\sigma} + \frac{\omega_N^2 \omega_{N-2}^1 - \omega_N^1 \omega_{N-2}^2}{\omega_N^1 (\omega_N^1 + 3\omega_N^2)} g_{\mu\nu} \delta_{\alpha\beta}^{\lambda\tau} g^{\rho\sigma} \\ &- \frac{\omega_{N-2}^1}{2\omega_N^1} g^{\lambda\tau} (g_{\mu\alpha} \delta_\beta^\sigma \delta_\nu^\rho + g_{\nu\alpha} \delta_\beta^\sigma \delta_\mu^\rho + g_{\mu\beta} \delta_\alpha^\sigma \delta_\nu^\rho + g_{\nu\beta} \delta_\alpha^\sigma \delta_\mu^\rho) \\ &- \frac{\omega_{N-2}^1 + 4\omega_{N-2}^2}{\omega_N^1} g_{\lambda\tau} g_{\alpha\beta} \delta_{\nu\mu}^{\rho\sigma} \end{aligned} \quad (30)$$

Using the Barvinsky-Vilkovisky trace formulae in (25), for flat background metric, we get the following expression for the divergences

$$\frac{i}{2} \text{Tr} \ln H_{\mu\nu,\alpha\beta} = \frac{1}{\varepsilon} \int d^4x \sqrt{-g} \text{ tr} \left(-\frac{1}{12} U^{\lambda\tau\rho\sigma} g_{\lambda\tau\rho\sigma}^{(2)} + \frac{1}{1920} V^{\lambda\tau\rho\sigma} V^{\lambda'\tau'\rho'\sigma'} g_{\lambda\tau\rho\sigma\lambda'\tau'\rho'\sigma'}^{(4)} \right), \quad (31)$$

where $\varepsilon = (4\pi)^2 (n-4)$ is the parameter of the dimensional regularization, and [37]

$$\begin{aligned} g_{\lambda\tau\rho\sigma}^{(2)} &= g_{\lambda\tau} g_\sigma + g_{\lambda\rho} g_{\tau\sigma} + g_{\lambda\sigma} g_{\rho\tau} \\ g_{\lambda\tau\lambda'\tau'\rho\sigma\rho'\sigma'}^{(4)} &= g_{\lambda\tau} g_{\lambda'\tau'} g_{\rho\sigma} g_{\rho'\sigma'} + \text{permutations}. \end{aligned}$$

Now, taking into account (29)–(31) we calculate the one-loop divergences for the theory on flat background. The result is

$$\Gamma_N^{div} = \frac{1}{\varepsilon} \int d^4x \sqrt{-g} \left[-\frac{2}{\omega_N^1 (\omega_N^1 + 3\omega_N^2)} u(N) + \frac{1}{(\omega_N^1)^2 (\omega_N^1 + 3\omega_N^2)^2} v(N) \right] \quad (32)$$

where

$$u(N, N > 1) = (6\omega_N^1 \omega_{N-2}^1 + 15\omega_N^2 \omega_{N-2}^1 + 3\omega_{N-2}^2 \omega_N^1)$$

$$v(N) = (\omega_N^1)^2(\omega_{N-1}^1 + 3\omega_{N-1}^2)^2 + 5(\omega_{N-1}^1)^2(\omega_N^1 + 3\omega_N^2)^2. \quad (33)$$

For the case $N = 1$ one needs to change the expression for U in (30).

$$\begin{aligned} U_{\mu\nu,\alpha\beta}^{\lambda\tau\rho\sigma}(N=1) &= g^{\lambda\tau}\omega_{-1} \left[-\frac{1}{\omega_1^1} \delta_{\mu\nu,\alpha\beta} g^{\rho\sigma} + \frac{\omega_1^1 + 2\omega_1^2}{2\omega_1^1(\omega_1^1 + 3\omega_1^2)} g_{\mu\nu} g_{\alpha\beta} g^{\rho\sigma} \right. \\ &\quad - \frac{(\omega_1^1 + 2\omega_1^2)}{2\omega_1^1(\omega_1^1 + 3\omega_1^2)} g_{\mu\nu} \delta_{\alpha\beta}^{\rho\sigma} - \frac{1}{\omega_1^1} g_{\alpha\beta} \delta_{\nu\mu}^{\rho\sigma} \\ &\quad \left. + \frac{1}{2\omega_1^1} (g_{\mu\alpha} \delta_{\beta}^{\sigma} \delta_{\nu}^{\rho} + g_{\nu\alpha} \delta_{\beta}^{\sigma} \delta_{\mu}^{\rho} + g_{\mu\beta} \delta_{\alpha}^{\sigma} \delta_{\nu}^{\rho} + g_{\nu\beta} \delta_{\alpha}^{\sigma} \delta_{\mu}^{\rho}) \right] \end{aligned} \quad (34)$$

Then, after inserting this expression into the formula (31) we also obtain (32) but with a different coefficient $u(N=1)$.

$$u(N=1) = \frac{3}{2} \omega_{-1} (3\omega_1^1 + 10\omega_1^2) \quad (35)$$

This formulae (32), (33) and (34) complete the calculation of the cosmological constant counterterm in an effective gravity theory S_{eff} . For $N > 2$ the above expressions are exact since there are no any additional divergences for the cosmological constant at higher loops. It is important to recall that the result is independent on the choice of the gauge fixing condition and on the parametrization of the quantum field. In this respect, it differs from the counterterms which appear in quantum gravity with two derivatives [38] and four derivatives [26, 28]. Thus adding the higher derivative terms to the effective theory introduces some relevant difference from this viewpoint.

It should be also interesting to calculate the divergence of the coefficient of the Einstein term R , because the corresponding beta function $\beta_G = \beta\omega_{-1}$ describes the running of the gravitational constant with the change of energy scale. The formulas (25) and (26) enable one to perform such a calculation for any finite N , but it would require an effort beyond our present possibilities. By dimensional arguments the counterterm which is linear in curvature has to be a linear combination of the coefficients $\omega_{N-1}^1, \omega_{N-1}^2$. We know that all the coefficients entering in this linear combination will give a factor $\omega_N^{1-1} (\omega_N^1 + 3\omega_N^2)^{-1}$, but the exact value requires explicit calculation. The renormalization of the parameters ω_0^1, ω_0^2 of the four derivative terms (13) can be also obtained from (25) and (26). They are bilinear in $(\omega_N^1, \omega_N^2, \dots)$ and contain the factor $(\omega_N^1)^{-1} (\omega_N^1 + 3\omega_N^2)^{-1}$.

The logarithmic divergent contribution to the renormalization of the cosmological constant (32) yields a renormalization group equation which governs its dependence on the energy scale. The corresponding beta-function is

$$\beta_{\omega_{-2}} = \frac{1}{(4\pi)^2} \left[-\frac{2u(N)}{\omega_N^1(\omega_N^1 + 3\omega_N^2)} + \frac{v(N)}{(\omega_N^1)^2(\omega_N^1 + 3\omega_N^2)^2} \right]. \quad (36)$$

One interesting consequence of this general analysis is that it provides different scaling scenarios for the running of the cosmological constant (as well as for the other relevant couplings). In principle it is possible to impose more constraints on those coefficients to suppress all logarithmically divergent contributions. In such a case the corresponding theory is finite and all beta functions vanish. But this behaviour only holds for energies higher than the effective scale of the theory. When the energy moves to lower values the leading ultraviolet terms with $2N + 4$ derivatives become irrelevant and new scaling scenarios (36) governed by the terms with $2N+2$ derivatives emerge. This can be seen from our calculations taking the limit when the leading ultraviolet coefficients tend to zero. We recover in such a case the beta function corresponding to the subleading terms. In this way the theory provides a hierarchy of different scaling scenarios from very short distances to cosmological distances governed by different beta functions. However, properly speaking to implement such multiple scale scenario we will need a set of different dimensionful parameter scales μ_1, \dots, μ_N . One interesting feature of this scenario is the existence of crossover between the different scaling windows appears. The renormalization group flow runs from ultraviolet fixed points to infrared fixed points step by step in a continuous non-linear way.

In this sense an open approach to quantum gravity it is possible, although the compatibility with closed approaches coming from some fundamental theories is also possible by imposing the corresponding constraints for the different parameter of the effective action S_{eff} . In particular, if a non-trivial fixed point is found the possibility of having a non-perturbative approach to quantum gravity is open. The theory defined by scaling limit around this point could have unexpected non-perturbative effects.

4. Structure of mass poles

In the previous section we have seen that the high derivative theory provides an interesting smooth approach to quantum gravity. In this framework the effective theory can exhibit interesting features like the vanishing of the coupling constant in the ultraviolet regime. Moreover, the theory provides a series of different scaling scenarios for all relevant physical quantities governed by different renormalization group fixed points.

However, as it is well known such a versatility of high derivatives theories implies serious problems from an unitarity viewpoint. In particular, the theory contains a plethora of unphysical massive ghosts. The analysis of this problem in the framework of string effective models [7, 8] have shown that the massive ghosts disappear from the spectrum in a special parametrization of the metric $g_{\mu\nu}$ (the background metric in its target space), while from string theory point of view this parametrization does not differ from the others [9].

Recently, it was suggested that in theories with more than two derivatives of the metric it could be possible to find a ghost scenario with only one ghost which would be the most massive fundamental particle of the theory having negative norm states in the Hilbert space,

whereas the other lighter fundamental particles would have positive norm states which are compatible with unitarity [39]. In fact, in the theories of the type considered in the present paper the structure of the (euclidean) propagator in the spin-2 and spin-0 reads

$$G(k) = \left(l_{2N+4} k^{2N+4} + l_{2N+2} k^{2N+2} + l_{2N} k^{2N} + \cdots + l_2 k^2 \right)^{-1} \quad (37)$$

where l_i are real numbers related with the values of ω_i^1 , ω_i^2 coefficients in (27). The expression (37) can be decomposed in terms of simple propagators as

$$G(k) = \frac{A_0}{k^2} + \frac{A_1}{k^2 + m_1^2} + \frac{A_2}{k^2 + m_2^2} + \cdots + \frac{A_{N+1}}{k^2 + m_{N+1}^2} \quad (38)$$

where the masses m_j^2 can be real or complex depending on the values of the coefficients l_i (37). The complex masses are always grouped in conjugate pairs. The idea introduced in Ref. [39] was based on the assumption that for some physical values of l_i all the mass poles m_j^2 are real and positive, $0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{N+1}^2$, and the coefficients A_r are all positive $A_r > 0$ for $r = 0, 1, \dots, N$, except the last one A_{N+1} which is negative $A_{N+1} < 0$. In this case only the heaviest particle would be an unphysical ghost, whereas all the others would be ordinary massive particles with positive energy. Thus, the effective theory would be unitary only till energies of the order of the mass of the heaviest unphysical particle m_{N+1}^2 , although this mass can be chosen much larger than the masses of other particles. Beyond such a scale the theory becomes unphysical. Unitarity can only be completely restored for all energy scales in the ultimate fundamental theory.

Unfortunately, this scheme can not work, although from a theoretical viewpoint was very appealing. The problem is that the assumptions concerning the behaviour of the massive poles m_i^2 , and their residua A_j in the high derivative theory are not consistent. The reason is that (see Appendix A for details) for any real monotone sequence of masses $0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{N+1}^2$, the signs of the corresponding residua alternate, i.e. $\text{sign}[A_j] = -\text{sign}[A_{j+1}]$. This can be easily understood from the basic property of any polynomial with real coefficients and real zeros which establishes that the signs of the slopes at the zeros do always alternate between consecutive zeros. Hence the physical assumption made in Ref. [39] is never satisfied for real masses. In the case of complex poles we have an even more pathological situation. Complex masses $m^2 < 0$ lead to pairs of unphysical tachyons and for the remaining real masses the above argument show that they correspond to alternating pairs of particles and ghosts. However, this pathological ghost masses distribution does not exclude the existence of a spectrum changing field reparametrization mapping the theory into an unitary theory [40].

In any case the dynamical role of unphysical massive ghosts only appears at energies of Planck order [20]. At very low energies (corresponding to the macroscopic length) they do not propagate, and the only relevant excitation is that related with the massless graviton.

In particular, all infrared quantum effects of the high derivative theory are the same as in Einstein gravity. This is specially relevant for quantities with singular infrared behaviour, e.g. long range correlation functions. For instance, if we evaluate the one loop quantum corrections to the gravitational potential, we get the same result as Donoghue [6], independently of the details of the higher derivative theory we consider. This remarkable result is due to the universality of the non-local singular infrared contributions to correlation functions [41]. Now, in the pure Einstein theory due to the absence of a mass term this singular IR behaviour extends to the ultraviolet and in fact can be easily computed in the UV regime, where it is linked with the divergent behaviour of the counterterms. The relation is similar to the one which exists between the UV counterterms and the beta functions of the coupling constant. Now, the UV behaviour of the higher derivative gravity is completely different and it is obviously dependent on the details (coefficients and number of derivatives) of the effective action. The renormalization group has in such a case a non-linear behaviour which goes from the particular UV behaviour associated to our regularized theory to the universal infrared Einstein regime. Between these two regimes we have momenta domains where new regimes with different scaling behaviour appear provided the masses of the different massive ghosts of the theory are widely separated. The nonlinear behaviour of the renormalization group allows to go from the UV to the infrared through this series of unstable regimes. The relevant result is that the long distance behaviour of the theory is the same as Einstein gravity with some small quantum corrections [6]. Indeed the contributions of the higher derivative terms become more and more relevant as we increase the range of energies or what it is the same we go to shorter distances. One of their effects is the generation of a running of the gravitational constant with the change of energy scale, and a running of the cosmological constant which can become stable at short distances due to the vanishing of the corresponding counterterms.

5. Interaction with matter fields

Let us now explore the interaction of high derivative gravity with matter and gauge fields. In particular, it is interesting to analyze how the quantum effects of this theory can change the scaling properties of the matter field correlation functions and the effective potential of the Higgs fields. For that purpose we introduce a general gauge model containing spinor, vector and scalar Higgs fields with gauge, Yukawa and scalar interactions. The total action has the form

$$S_{\text{total}} = S_{\text{eff}} + S_{\text{gauge}} \quad (39)$$

where S_{eff} denotes the action of our high derivative gravity (1) and S_{gauge} is the action of the gauge model. We assume, as in the previous sections, that the gravitational propagator has an ultraviolet behaviour of order $\mathcal{O}(k^{-(2N+4)})$.

The diagrams contributing to any correlation function with external matter lines split into two sets. Diagrams with only matter internal lines, and diagrams with an internal gravitational propagators. The contributions of the diagrams of the first type (e.g. diagrams of the Figure 1) are equal to the contributions of the gauge model in a classical gravitational background. It can be shown [30] (and references therein) that in general the corresponding beta-functions for the matter fields couplings (gauge, Yukawa and scalar) are exactly the same as in flat space-time. The novel aspects are the need of the non-minimal coupling $\xi R\phi^2$ for every scalar field ϕ , the renormalization of ξ and the appearance of the corresponding beta-function β_ξ [30].

Indeed, there also appear additional contributions to the renormalization of the parameters ω_0^1 , ω_0^2 , ω_{-1} , ω_{-2} generated by the loops of matter fields in the diagrams with only gravitational external lines (e.g. diagram (2) of Figure 1). Diagrams involving only internal and external gravitational lines are only divergent when the inequality (12) holds. Both types of divergent counter-terms are of the form (13).

Let us now consider the most general diagrams which have some external matter lines and any kind of internal lines (Figure 2). Here one has to consider the cases $N > 1$ and $N = 1$ separately. For $N > 1$ all such diagrams are finite by power counting, thus they do not contribute to the beta-functions for the matter field couplings (gauge, Yukawa, scalar), which are actually the same as in the theory without quantum gravity. For gauge coupling constants the result is even more general. Their beta-function are not affected by any kind of the gravitational interaction. In the Einstein gravity it was shown in Ref. [3]. The effect is essentially based on the non-renormalizability of the Einstein-Maxwell or Einstein-Yang-Mills system (power counting does not permit any gravitational contribution to $(F_{\mu\nu}^a)^2$). In the fourth derivative quantum gravity the gravity-independence of the gauge coupling has been found in Ref. [26] as a result of unexpected cancellation of two diagrams. What we have shown is that the same result holds for the general theory (1) with $N > 0$. In summary, asymptotic freedom is stable under quantization of gravity.

In the case $N = 1$, there appear two new divergent diagrams with external scalar lines and internal graviton loops (see Figure 3). The divergence of these diagrams is generated by the nonminimal interaction term in the scalar (Higgs) sector of the gauge model. In fact the divergences generated by such mixed diagrams are always universal, i.e. they do not depend on the gauge group or multiplet composition. Consider, for simplicity, one real scalar field.

$$S_{sc} = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \xi R \phi^2 - \frac{f}{24} \phi^4 \right) \quad (40)$$

Diagrams of Figure 3 can generate UV divergences only if the two derivatives introduced by the scalar curvature in $\frac{1}{2} \xi R \phi^2$ term act to the internal gravitational lines. Therefore they can only affect the renormalization of the mass m^2 but none of the coupling constants.

The corresponding mass counterterm is (see Appendix C)

$$\Gamma_{\text{scalar}}^{\text{div}} = \frac{1}{\varepsilon} \frac{3\xi}{\omega_1^1 + 3\omega_1^2} \left(\frac{3\omega_1^1 + 10\omega_1^2}{\omega_1^1} - \xi \right) \int d^4x \sqrt{-g} \phi^2.$$

This divergence generates a gravitational correction to the known beta-function for the scalar mass, which appears due to the gauge, Yukawa and scalar interactions

$$\begin{aligned} \beta_{m^2}^{\text{total}} &= \beta_{m^2}^{\text{known}} + \delta^{\text{grav}} \beta_{m^2} \\ \delta^{\text{grav}} \beta_{m^2} &= \frac{1}{16\pi^2} \frac{3}{(\omega_1^1 + 3\omega_2^2)} \xi \left(\xi - \frac{3\omega_1^1 + 10\omega_1^2}{\omega_1^1} \right) \end{aligned} \quad (41)$$

In summary, in the framework of the higher derivative model of quantum gravity the beta functions for the gauge, Yukawa and scalar self-coupling are not affected by quantum gravity corrections. The only quantum effect is the correction to the mass beta-function of the scalar field (41) which only appears for the case $N = 1$.

We remark that in fourth derivative quantum gravity the contributions of gravitational loops to the beta-functions for the Yukawa and scalar couplings are nontrivial [29], they can even change the asymptotic behaviour of such theories (see also [30]). In contrast, for $N > 0$ there is no divergent contribution of gravity to the matter or gauge fields sector.

6. Conclusions

The above results show that in the high derivative approach to quantum gravity the UV behaviour is so smooth that its corrections to gauge theories do not modify the scaling UV behaviour of gauge and matter correlation functions for $N > 0$. However, in general, the pure gravitational selfinteraction is not completely self-regularized. As we have shown one-loop divergences are generically of the same type (13) that those of the Einstein theory. This is a general feature of non-abelian gauge theories. The analysis of this one-loop divergences shows that they can be cancelled by any of the following prescriptions:

- i) Pauli-Villars covariant regularization
- ii) Fine tuning of the high derivative couplings

The theory with a finite number of couplings can be considered either as a regularization of quantum gravity or better as an effective theory of quantum gravity. As effective theory some of its infrared properties, like the corrections to the Newtonian potential, are universal and do not depend on the UV behaviour of the theory. We have shown that the structure of mass poles is such that the ghost masses are intercalated between those of particles in such

a way that does not allow to recover unitarity unless we restrict to energies below the first mass scale of the theory.

Another interesting feature of the high derivative approach is that the UV scaling regime is independent of the gauge parameter and the parametrization of metric fields.

In the cases where the theory is finite for all momentum scales it might be possible, in principle, to find interesting non-perturbative scaling regimes which shall give rise to novel non-perturbative approaches to quantum gravity.

In summary, the high derivative approach offers a good selfconsistent framework to deal with quantum gravity effects for intermediate scales between low energy phenomenology and high energy fundamental unified theories.

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Appendix A

Let us prove now the intercalation property of particles an ghost masses which have been used in the main text. We consider for simplicity the euclidean formalism. The euclidean propagator $G(k^2)$ has the form

$$G(k^2) = \frac{l_{2N+4}^{-1}}{\prod_{i=0}^{N+1}(k^2 + m_i^2)} \quad (A.1)$$

where the masses m_j , $j = 0 \cdots N + 1$ are functions of the coefficients l_{2j+2} ; and can be rewritten in the form (38). Therefore, $G(k)$ has simple poles in $k^2 = -m_j^2$, $j = 0 \cdots N + 1$ if all the masses are different.

In the complex plane \mathbb{C} , $G(z)$ is a meromorphic function with $N + 2$ simple poles. Let Γ_j be a closed path in \mathbb{C} around the pole $-m_j^2$ not encircling any other pole. Then, from (A.1), we have, for $j = 0 \cdots N + 1$

$$\oint_{\Gamma_j} G(z) = 2\pi i \text{Res} \left[\frac{l_{2N+4}^{-1}}{\prod_{i=0}^{N+1}(z + m_i^2)}, -m_j^2 \right] = 2\pi i l_{2N+4}^{-1} \prod_{i=0, i \neq j}^{N+1} \frac{1}{(m_i^2 - m_j^2)} \quad (A.2)$$

and from (38)

$$\oint_{\Gamma_j} G(z) = 2\pi i \text{Res} \left[\sum_{i=0}^{N+1} \frac{A_i}{(z + m_i^2)}, -m_j^2 \right] = 2\pi i A_j, \quad j = 0 \cdots N + 1 \quad (A.3)$$

Then, from (A.2) and (A.3) we get

$$A_j = l_{2N+4}^{-1} \prod_{i=0, i \neq j}^{N+1} \frac{1}{(m_i^2 - m_j^2)}, \quad j = 0 \cdots N+1 \quad (A.4)$$

When the masses m_j are real or purely imaginary, their squares m_j^2 are real, and once they are ordered ($m_i^2 < m_{i+1}^2 \forall i = 0 \cdots N$), (A.4) tells us that the signs of the residua alternate, i.e. $\text{sign}[A_j] = (-1)^j \text{sign}[l_{2N+4}^{-1}]$, $j = 0 \cdots N+1$.

This result can also be understood in pure algebraic terms. If the polynomial $G(k^2)^{-1}$ of k^2 which defines the propagator only has non-degenerate real zeros (and this is the assumption we have made) the signs of the slopes at two consecutive zeros alternates.

On the other hand, if one m_j^2 is complex, there is another one m_l^2 which is its complex conjugate, $m_l^2 = m_j^{2*}$. We can easily show that their residua A_j, A_l are also pairs of complex conjugate numbers $A_j = A_l^*$, whereas the residua A_j corresponding to the remaining real masses m_j^2 have alternating signs.

Appendix B

Let us prove in detail the gauge fixing independence of the one-loop divergences by using the method of [37] (see also [28, 35] for the fourth derivative gravity).

Diffeomorphism invariance implies that the classical action is invariant under the infinitesimal transformation

$$\delta g_{\mu\nu} = \mathcal{D}_{\mu\nu,\lambda} \xi^\lambda, \quad \mathcal{D}_{\mu\nu,\lambda} = -(g_{\mu\lambda} \nabla_\nu + g_{\nu\lambda} \nabla_\mu)$$

which leads to the Noether identity

$$\mathcal{D}_{\mu\nu,\lambda} \frac{\delta S}{\delta g_{\mu\nu}} = 0 \quad (B.1)$$

The gauge fixing term (7) introduces a number of gauge parameters $\sigma_i, \gamma_i, \beta$. Let us generically denote them by σ . The propagators $G^{\mu\nu,\alpha\beta}, \Omega^\alpha{}_\beta$ of the metric field $h_{\mu\nu}$ and of the Faddeev-Popov ghosts $\bar{c}_{\beta_1}, c^{\beta_2}$ are defined as usual by the relations

$$K_{\mu\nu,\alpha\beta} G^{\alpha\beta,\rho\sigma} = \delta_{\mu\nu}{}^{\rho\sigma}, \quad M^\alpha{}_\beta \Omega^\beta{}_\lambda = \delta_\lambda^\alpha \quad (B.2)$$

where

$$K_{\mu\nu,\alpha\beta} = \frac{\delta^2 S}{\delta g_{\mu\nu} \delta g_{\alpha\beta}} + \frac{\delta \chi_\rho}{\delta g_{\mu\nu}} C^{\rho\sigma} \frac{\delta \chi_\sigma}{\delta g_{\alpha\beta}}$$

and

$$M^\alpha{}_\beta = C^{\alpha\gamma} \frac{\delta \chi_\gamma}{\delta g_{\mu\nu}} \mathcal{D}_{\mu\nu,\beta} \quad (B.3)$$

From (B.1) and (B.3) it follows the Ward identity which links the two propagators and the gauge fixing functional

$$\mathcal{D}_{\mu\nu,\beta} \Omega^\beta{}_\alpha - G_{\mu\nu,\rho\sigma} \frac{\delta\chi_\alpha}{\delta g_{\rho\sigma}} = -G_{\mu\nu,\rho\sigma} \frac{\delta\mathcal{D}_{\lambda\tau,\gamma}}{\delta g_{\rho\sigma}} \frac{\delta S}{\delta g_{\lambda\tau}} \Omega^\gamma{}_\alpha \quad (B.4)$$

The one loop effective action Γ is given by the expression

$$i\Gamma = -\frac{1}{2} \text{Tr} \ln K_{\mu\nu,\alpha\beta} + \text{Tr} \ln M_\alpha{}^\beta - \frac{1}{2} \text{Tr} \ln C^{\alpha\beta} \quad (B.5)$$

Notice that we use a different definition of M and that is why the sign of $\text{Tr} \ln C$ in (B.5) is opposite to that of expression (18). If we denote by \dot{X} the derivative of the quantity X with respect to a generic parameter introduced by the gauge fixing functional, then the derivative of (B.5) with respect of such a parameter can be estimated with the use of (B.4). The result is

$$i\dot{\Gamma} = -\frac{1}{2} \text{Tr} G_{\mu\nu,\rho\sigma} \frac{\delta\mathcal{D}_{\lambda\tau,\beta}}{\delta g_{\rho\sigma}} \frac{\delta S}{\delta g_{\lambda\tau}} \Omega^\beta{}^\alpha \left[\frac{\delta\chi_\gamma}{\delta g_{\mu\nu}} \dot{C}^{\alpha\gamma} + 2C_{\alpha\gamma} \frac{\delta\dot{\chi}^\gamma}{\delta g_{\mu\nu}} \right] \quad (B.6)$$

The difference between the one-loop corrections in two different gauges, $\Gamma(\sigma) - \Gamma(\bar{\sigma})$ is obtained by integrating (B.6) for all the gauge fixing parameters from σ to $\bar{\sigma}$ [35]. The equation (B.6) gives the general form of the gauge parameters dependence of the one-loop effective action which is proportional to the motion equations, $\delta S/\delta g_{\alpha\beta}$. For the higher derivative theory with $N > 0$ this implies the independence of the one loop divergent contributions on the gauge parameters σ_n, γ_n and β as proved in section 3.

Appendix C

The divergent part of the diagrams of Figure 3 can be calculated by means of the background field method and Schwinger-DeWitt technique, using the generalization introduced in Ref. [37] and the methods developed in Ref. [29] (see also [30]). In the present theory the calculation is technically simpler than the one carried out in [29]. First, we notice that, by power counting, we only need to calculate the divergent terms of the form $Z\phi^2$ where ϕ is the background scalar, and Z is a divergent coefficient with dimension of mass². Besides the splitting of the metric $g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}$ into background and quantum fluctuations we have a similar splitting for the scalar field $\phi \rightarrow \phi + \varphi$. We can therefore consider the background field ϕ as a constant. The second diagram contains purely gravitational loops, and it can be evaluated with the algorithm used in (32). Then one gets the result by replacing

$$\omega_{-1} \longrightarrow \omega_{-1} - \frac{1}{2}\xi\phi^2 \quad (C.1)$$

in the expression (35).

The calculation of the divergences generated by the first diagram is more involved. Power counting tells us that this contribution only appears when all derivatives of the vertices of those diagrams act on the internal lines. In the framework of the background field method this means that we need to keep only the terms with two derivatives of the quantum fields when performing the background field expansion of the $\xi R\phi^2$ term in (40). On the other hand one can neglect all the non-leading order terms in the pure gravitational $h - h$ sector and consider only the contribution of the higher derivative terms, because all the others do not contribute to the divergent counterterms. The same feature occurs for non-leading derivative terms in the scalar $\phi - \phi$ sector. However the higher order terms in the mixed $\phi - h$ and $h - \phi$ sectors are relevant. In summary, it can be shown that all the divergences are contained in the expression $(i/2) \operatorname{Tr} \ln \hat{H}$ where

$$\hat{H} = \begin{pmatrix} A_{\mu\nu,\alpha\beta} \square^3 & [P^{\lambda\tau}]_{\mu\nu} \nabla_\lambda \nabla_\tau \\ [Q^{\lambda\tau}]_{\alpha\beta} \nabla_\lambda \nabla_\tau & -\frac{1}{2} \square \end{pmatrix}, \quad (C.2)$$

and A, P, Q are given by

$$A_{\mu\nu,\alpha\beta} = \frac{\omega_1^1}{4} \left(\delta_{\mu\nu,\alpha\beta} - \frac{\omega_1^1 + 4\omega_1^2}{\omega_1^2} g_{\mu\nu} g_{\alpha\beta} \right)$$

$$[P^{\lambda\tau}]_{\mu\nu} = \frac{1}{2} \xi \phi \left(\delta^{\lambda\tau}_{\mu\nu} - g^{\lambda\tau} g_{\mu\nu} \right), \quad [\hat{Q}^{\lambda\tau}]_{\alpha\beta} = \frac{1}{2} \xi \phi \left(\delta^{\lambda\tau}_{\alpha\beta} - g^{\lambda\tau} g_{\alpha\beta} \right) \quad (C.3)$$

This formula can be compared with the similar expression associated to fourth derivative gravity coupled to matter [30].

Thus,

$$\operatorname{Tr} \ln \hat{H} = \operatorname{Tr} \ln \begin{pmatrix} A_{\mu\nu,\alpha\beta} & 0 \\ 0 & -1/2 \end{pmatrix} + \operatorname{Tr} \ln \begin{pmatrix} \delta_{\mu\nu,\alpha\beta} \square^3 & 0 \\ 0 & \square \end{pmatrix}$$

$$+ \operatorname{Tr} \ln \left[\begin{pmatrix} \delta_{\alpha\beta}^{\mu\nu} & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & [P^{\lambda\tau}]_{\mu\nu} \nabla_\lambda \nabla_\tau \square^{-1} \\ [Q^{\lambda\tau}]_{\alpha\beta} \nabla_\lambda \nabla_\tau \square^{-3} & 0 \end{pmatrix} \right] \quad (C.4)$$

The first term in the last expression is the logarithm of the determinant of a c -matrix, and thus it is finite. The second term depends only on the metric but not on the background scalar, and therefore it can give only contributions to the renormalization of the gravitational sector. Since this renormalization has been already considered in the diagrams of Figure 2, we have to ignore here its contributions. Expanding the last logarithm in power series, one can keep only the terms with proper background dimension. Power counting tells us that only these terms are divergent (one can also use universal trace formulae of [37] to check the convergence of the rest of the series). The $Z\phi^2$ -type divergence is given by the second term of the series and has the form

$$\frac{1}{12\varepsilon} \int d^4x \sqrt{-g} \ [P^{\lambda\tau}]_{\mu\nu} A_{\mu\nu,\alpha\beta} [Q^{\rho\sigma}]_{\alpha\beta} g_{\lambda\tau\rho\sigma}^{(2)} \quad (C.5)$$

Substituting the above expressions for A, P, Q and taking into account the contribution of the second diagram of Figure 3, we get the following divergent part of the effective action

$$\Gamma_{scalar}^{div} = \frac{1}{\varepsilon} \frac{3\xi}{\omega_1^1 + 3\omega_1^2} \left(\frac{3\omega_1^1 + 10\omega_1^2}{\omega_1^1} - \xi \right) \int d^4x \sqrt{-g} \phi^2 \quad (C.6)$$

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FIGURE CAPTIONS

Figure 1: Diagrams contributing to the renormalization of matter and gauge field (thick lines) interactions in a gravitational classical background (wavy lines).

Figure 2: Diagrams contributing to the renormalization of gravitational, matter and gauge field interactions including quantum gravity corrections.

Figure 3: Divergent diagrams contributing to the mass of scalar particles in the $N = 1$ theory. The divergent contribution is generated by the non-minimal couplings

$$\frac{1}{2}\xi \int d^4x\sqrt{-g} \phi^2$$

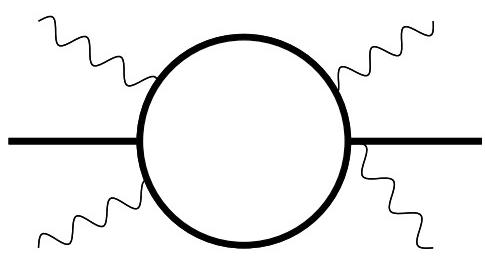


Diagram (1)

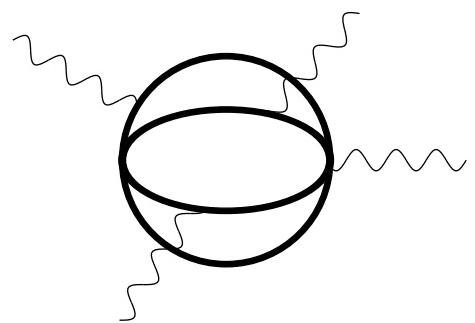


Diagram (2)

FIGURE 1

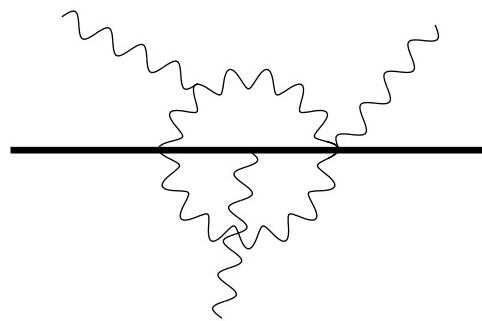


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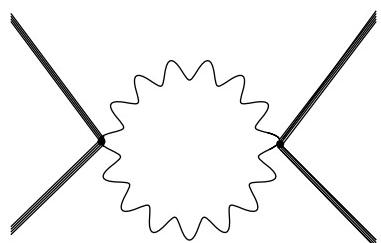


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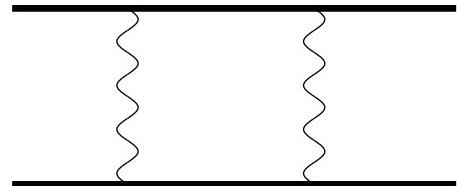


Diagram (3)

FIGURE 2

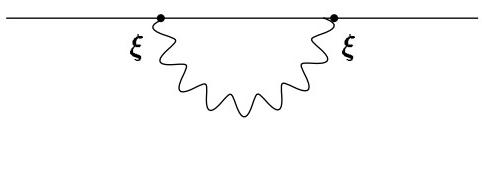


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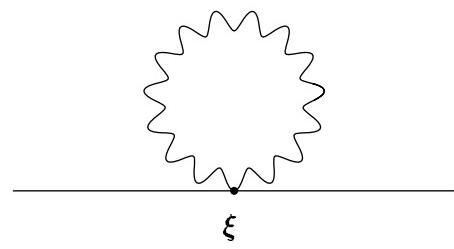


Diagram (2)

FIGURE 3